

## Section A: Pure Mathematics [40 marks]

1 The curve  $C$  has equation  $y = \frac{x^2 + ax + 1}{x + b}$ .

- (i) Given that the graph passes through the point  $(-1, 0)$  and has a vertical asymptote at  $x = 2$ , find the values for  $a$  and  $b$ . [2]
- (ii) Sketch the graph of  $C$ , clearly stating the equations of any asymptotes and the coordinates of any axial intercepts and/or turning points. [3]
- (iii) Hence, find the range of values of  $k$  for which the equation  $[(x+1)(x+k)]^2 = x-2$  has two real roots, where  $k \in \mathbb{Z}$ . [3]

2 The functions  $f$  and  $g$  are defined by:

$$f : x \mapsto \sqrt{x^2 + 4x - 3}, \quad x \geq 1$$

$$g : x \mapsto \frac{12}{1 - x^2}, \quad x \neq -1, 1$$

- (i) Show that  $f$  is one-one. [1]
- (ii) Find  $f^{-1}$  in a similar form. [3]
- (iii) Find the largest domain of  $g$  such that  $fg$  exists. [2]
- (iv) Hence find the range of  $fg$ . [3]

3 The complex numbers  $z$  and  $w$  are such that

$$|z - 2i| = 1$$

$$\arg(w) = k$$

- (i) Given  $k = \frac{\pi}{4}$ , sketch the loci on an Argand diagram. Give the geometrical description of the loci. [4]
- (ii) Find the values of  $k$  such that the loci intersect at exactly one point. [3]
- (iii) Given that  $k = \frac{5\pi}{6}$ , find the values of  $w$  and  $z$  in the form  $x + iy$  that minimize  $|w - z|$ . [4]

- 4 (a) Show that the differential equation

$$x \frac{du}{dx} + u - \sqrt{4 - (ux)^2} = 0$$

may be reduced by means of the substitution  $y = ux + 2$  to

$$\frac{dy}{dx} = \sqrt{4y - y^2}. \quad [2]$$

Hence, find the general solution of the differential equation, leaving your answer in exact form. [4]

- (b) The displacement  $s$  (metres) of an object moving in a straight line from a fixed point  $O$  is related to time  $t$  (seconds) by the differential equation

$$\frac{ds}{dt} = \sqrt{4s - s^2}.$$

- (i) Sketch the solution curve of the particular solution for  $0 \leq t \leq 4\pi$  given that

$$s = 1 \text{ when } t = \frac{5\pi}{6}. \quad [4]$$

- (ii) Describe the motion of the object and comment on whether the differential equation in  $s$  and  $t$  is an appropriate model in the real-life context. [2]

### Section B: Statistics [60 marks]

- 5 A committee consists of  $i$  representatives from faculty  $I$ ,  $j$  representatives from faculty  $J$  and  $k$  representatives from faculty  $K$ . Find the number of ways of arranging the members of the committee

- (i) in a circle for a meeting; [1]  
 (ii) in a row with representatives from the same faculty seated together. [2]

- 6 (a) Explain briefly the difference between a sample and a population. [2]  
 (b) A school has 120 teachers belonging to the following four departments:

Humanities	21
Language	33
Mathematics	27
Science	39

The Principal wants to gather feedback from the staff regarding some school programmes. A sample of 40 teachers from the four departments is to be chosen to take part in a survey.

- (i) The sample is formed by randomly selecting 10 teachers from each of the four departments. Explain why this procedure will not result in a random sample of teachers. [2]  
 (ii) Describe how a stratified random sample of the 40 teachers may be obtained. [2]

- 7** In conjunction with SG50 celebratory events, a survey is administered to find out people's knowledge of Singapore's history. A person surveyed has to answer 8 multiple-choice questions, each with the same number of options given and has only 1 correct answer. Suppose that for someone totally clueless of Singapore's history, there is a probability of 0.503 (correct to 3 significant figures) to get at most 1 correct answer when doing the survey, show that each multiple-choice question has 5 options. [2]

Assume that foreigners are totally clueless of Singapore's history. Find the most probable number of correct answers obtained by a foreigner who did the survey. [2]

A sample of 50 foreigners who did the survey is taken. State with a reason, the distribution for the mean number of correct answers obtained. Hence, find the probability that the mean number of correct answers is not less than 2. [3]

- 8** A hospital receives calls requiring ambulance assistance.

- (i) State one condition needed for the number of calls in a day requiring ambulance assistance to be well modelled by a Poisson distribution. [1]

Assume that the hospital receives on average 1 call a day which requires ambulance.

- (ii) Find the probability that, in a week consisting 7 days, there are less than 5 calls requiring ambulance assistance. [2]
- (iii) Use a suitable approximation to find the probability that, in a year with 52 weeks, more than 40 but at most 45 weeks have more than 4 calls requiring assistance. [4]

- 9** The time required by a machine to complete a task is normally distributed and believed to have mean 47.0 seconds. A sample of 12 timings is taken and is found to have mean 48.1 seconds and standard deviation 1.9 seconds. A test is carried out, at the 3% significance level, to determine whether the mean time required by the machine to complete the task is understated.

- (i) State appropriate hypotheses for the test, defining any symbols you use. [2]
- (ii) Carry out the test. [4]
- (iii) It is subsequently found that the standard deviation for the time required by the machine to finish the task is 2.1 seconds. Find the range of values of the sample mean for which the null hypothesis would be rejected. [2]

**10** A box contains 2 red balls, 2 blue balls, 2 green balls and 2 yellow balls. Balls of the same colour are numbered 0 and 1. The balls are indistinguishable except for their colours and numbers. Three balls are drawn from the box at random, one by one and with replacement. Find the probability that

- (i) all of them have different colours; [2]  
 (ii) the sum of their numbers is even, given that they all have different colours. [4]

Events  $A$  and  $B$  are defined as follows.

$A$ : the sum of the numbers on the 3 balls is even

$B$ : the 3 balls all have different colours

Determine if  $A$  and  $B$  are independent. [2]

**11** The masses, in grams, of mobile phones sold by companies  $A$  and  $B$  have independent normal distributions with means and standard deviations as shown in the table.

	Mean weight	Standard deviation
Company $A$	130	6
Company $B$	$\mu$	$\sigma$

- (i) The mass of a random mobile phone sold by Company  $B$  is equally likely to be less than 134 g and more than 146 g, with a probability of 0.234. Find  $\mu$  and  $\sigma$ . [3]

For the rest of the question, use  $\mu = 136$  and  $\sigma = 8$ .

- (ii) Find the probability that the mass of a randomly chosen mobile phone sold by Company  $A$  is within 150 g of twice the mass of a randomly chosen mobile phone sold by Company  $B$ . [3]  
 (iii) The masses of 10 mobile phones sold by Company  $A$  are taken. Find the probability that the heaviest of them is not more than 135 g. [2]  
 (iv) State an assumption needed for your calculation in part (iii). [1]

- 12** (a) The equation of the estimated least squares regression line of  $y$  on  $x$  for a set of bivariate data is  $y = a + bx$ . Explain what do you understand by the least squares regression line of  $y$  on  $x$ . [2]
- (b) A decaying radioactive material is known to emit radioactive waves. The intensity of the radioactive emission  $I$  for time  $t$ , in appropriate units, is recorded. The results are shown in the table below.

Time, $t$	1	2	3	4	5	6	7
Intensity of emission, $I$	15	8.8	4.6	3.3	1.8	1.5	0.9

- (i) Draw a scatter diagram for these values, labelling the axes clearly. [2]
- (ii) Calculate the product moment correlation coefficient and explain why its value does not necessary mean that the best model for the relationship between  $I$  and  $t$  is  $I = a + bt$ , where  $a$  and  $b$  are constants. [2]

It is thought that the radioactive emission,  $I$ , can be modelled by one of the formulae

$$I = c + dt^2 \text{ or } I = e + f \ln t$$

where  $c$ ,  $d$ ,  $e$  and  $f$  are constants.

- (iii) Explain how you would determine which of  $I = c + dt^2$  or  $I = e + f \ln t$  is a better model. [1]
- (iv) It is required to estimate the value of  $t$  for which  $I = 2.5$ . Find the equation of a suitable regression line, and use it to find the required estimate. Comment on the reliability of your estimate. [5]

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