ANGLO-CHINESE JUNIOR COLLEGE
MATHEMATICS DEPARTMENT

MATHEMATICS
Higher 2 **9740 / 02**

Paper 2 26 August 2015

JC 2 PRELIMINARY EXAMINATION

Time allowed**: 3 hours**

Additional Materials: List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Index number, Form Class, graphic and/or scientific calculator model/s on the cover page.
Write your Index number and full name on all the work you hand in.
Write in dark blue or black pen on your answer scripts.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in the question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
At the end of the examination, fasten all your work securely together.

This document consists of **6** printed pages.

Anglo-Chinese Junior College

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ANGLO-CHINESE JUNIOR COLLEGE
MATHEMATICS DEPARTMENT
JC2 Preliminary Examination 2015

MATHEMATICS 9740
Higher 2
Paper 2

/ 100

Index No: | | | | | Form Class: _____

Name: _____

Calculator model: _____

Arrange your answers in the same numerical order.

Place this cover sheet on top of them and tie them together with the string provided.

Question No.	Marks
1	/9
2	/9
3	/11
4	/11
5	/3
6	/5
7	/7
8	/6
9	/9
10	/10
11	/10
12	/10

Summary of Areas for Improvement				
Knowledge (K)	**Careless Mistakes (C)**	**Read/Interpret Qn wrongly (R)**	**Formula (F)**	**Presentation (P)**

Section A: Pure Mathematics [40 marks]

1 **(i)** Differentiate $\frac{1}{2x^2+1}$ with respect to x. [1]

(ii) Hence find $\int \frac{x^2}{\left(2x^2+1\right)^2}\,\mathrm{d}x$. [4]

The curve C has equation $y=\frac{x}{2x^2+1}$, where $x \geq 0$.

(iii) The region bounded by C and the line $2y = x$ is rotated 2π radians about the x-axis. Find the exact volume of the solid obtained. [4]

2 The complex number z satisfies the equation $\left|\frac{z+\mathrm{i}-3}{2+\mathrm{i}z}\right|=1$.

(i) Using an algebraic method, find the purely imaginary number that satisfies the given relation. [2]

(ii) Sketch the locus of the points representing z, labelling the coordinates of the y-intercept. [2]

(iii) Describe the locus of the points representing w such that $|w-4\mathrm{i}|=a$, where a is a non-zero constant. Hence find the exact value of a such that there is exactly one value of z that satisfies $\left|\frac{z+\mathrm{i}-3}{2+\mathrm{i}z}\right|=1$ and $|z-4\mathrm{i}|=a$. [3]

For this value of a, find the exact value of z that satisfies the above conditions, giving your answer in the form $x+\mathrm{i}y$ where $x,\ y \in \mathbb{R}$. [2]

3 **(i)** Sketch the curve with equation $y=x+\frac{1}{x}$, $x \neq 0$, stating the equation(s) of any asymptotes and the coordinates of any turning points and any points of intersection with the axes. [3]

The functions f and g are defined as follows

$$\mathrm{f}: x \mapsto x-a+\frac{1}{x-a},\ x \geq b,\ x \neq a, \qquad \mathrm{g}: x \mapsto \frac{1}{\ln x},\ x>0,\ x \neq 1$$

where a and b are constants and $a>0$.

(ii) Using part **(i)** or otherwise, state the smallest value of b, in terms of a, such that f is one-one. [2]

(iii) Explain why the composite function gf exists and find the range of gf. [3]

(iv) Given that $a = 2$, define f^{-1} in a similar form. [3]

[Turn Over

4 Planes p_1 and p_2 have the following equations:

$$p_1: \quad 3x-2y+6z=2$$

$$p_2: \quad \mathbf{r}=(1+2s+2t)\mathbf{i}+(-2-3t)\mathbf{j}+(-s-2t)\mathbf{k}, \quad s,t\in\mathbb{R}$$

(i) Explain why p_1 and p_2 are parallel and distinct planes. Hence find the shortest distance between these two planes. [5]

(ii) The line l has equation $\mathbf{r}=(5+\beta\lambda)\mathbf{i}+(-5+3\lambda)\mathbf{j}+(\alpha+\lambda)\mathbf{k}$, where $\lambda\in\mathbb{R}$ and α, β are real constants.

(a) Find conditions on α and β such that l only intersects one plane but not the other. [3]

(b) The angle between l and p_1 is 22°. Find the two possible values of β. [3]

Section B: Statistics [60 marks]

5 At a railway station, serious delays occur at random such that the expected number of serious delays per week is 4.3 and the variance is 2.56. State, with a reason, whether the number of serious delays may be modelled by a Poisson distribution.
Find the probability that, in a period of 60 weeks, not more than 4 serious delays occur on average per week. [3]

6 In a junior college there are 750 Year 2 students in the science faculty. These students are grouped according to 4 major subject combinations, namely SA, SB, SC and SD. During a science talk, the students are seated according to combination in four different venues. The number of students in each combination and their venues are given in the table below.

Combination	SA	SB	SC	SD
Number of Year 2 students	195	180	225	150
Venue	LT1	LT2	LT3	LT4

The teacher-in-charge of this talk intends to obtain a sample of 100 students for a survey. She selects 25 students from the last occupied row in each venue for the survey.

(i) Name the sampling method described and state a reason, in the context of the question, why this sampling method is not desirable. [2]

(ii) Suggest a method of obtaining a more representative sample and describe how it may be carried out. [3]

7 **(a)** A teacher wants to set a class test for her class of 25 students. She plans to give each student the same questions, but have each student's questions appear in a different order. Find the least number of questions she must set. [2]

(b) Sally has 12 bars of chocolates in four different flavours as summarised in the table below.

flavour	white	milk	dark	hazelnut
number	4	3	4	1

Sally intends to give one bar of chocolate each to three of her best friends in school.

(i) Find the number of ways she can do it. [3]

(ii) Sally remembers that one particular friend amongst the three has a nut allergy, and should not be given the hazelnut chocolate bar. Find the number of ways she can now give her friends the chocolates. [2]

8 A particular brand of paper cups is found to have capacity that is normally distributed with mean 500 ml and standard deviation 45 ml.

(i) Andy and twenty of his friends are each given one paper cup to fill a 10-litre tank with water. Each person completely fills his paper cup with water once, and then empties the water into the tank. Find the probability that the tank is completely filled after the last person empties his cup. [2]

(ii) Andy decides to attempt to fill the tank with water using his one cup. He does so by making 21 trips from the tap to the tank, each time filling his cup completely at the tap before pouring the contents into the tank. Show that the probability that he manages to fill the tank is 0.70163, correct to 5 significant figures. [1]

(iii) In a game, each of 60 people is given one paper cup and one 10-litre tank to fill with water. If every person makes 21 trips from the tap to his tank, using a suitable approximation, find the probability that at most 45 of the tanks will be filled. [3]

9 John and Jane are working on a research project about the heights of 17-year-old boys from Taz, a minority ethnic community. They come across an internet website claiming that the average height of 17-year-old Taz boys is 170 cm.

(a) John believes that the average height quoted on the website is too low. He assumed that the heights of 17-year-old Taz boys follow a normal distribution and measured the heights of eight randomly chosen boys. The data collected, measured in cm, are as follows:

171.8 167.4 174.5 169.4 171 175.5 170.4 173.5

Carry out a test of John's belief at the 5% significance level, defining any symbols you used in the hypotheses. [4]

(b) Jane finds out that the standard deviation of the heights of 17-year-old Taz boys is 4.2 cm. She decides to carry out a one-tail test to determine whether the claim on the website is valid by measuring the heights of a random sample of n 17-year-old Taz boys. Their mean height is found to be 168.6 cm.

(i) State appropriate hypotheses for the test. [1]

(ii) Given that the null hypothesis is not rejected at 5% level of significance, find the range of values of n. State, giving a reason, whether it is necessary to assume that the heights of Taz 17-year old boys are normally distributed. [4]

10 A confectionary bakes chocolate chip cookies in batches. To each large batch of cookie dough, 600 chocolate chips are added and thoroughly mixed into the dough to make 150 cookies.

(i) State, in this context, two conditions for the number of chocolate chips in one cookie to be well modelled by a Poisson distribution. [2]

For the remainder of this question, assume that the conditions in **(i)** are met.

(ii) Find the probability that, out of four randomly chosen cookies from the confectionary, exactly two have five chocolate chips each and the other two have more than five chocolate chips each. [2]

(iii) The confectionary sells the cookies in boxes of 30 each. Using a suitable approximation, find the probability that there are at least two cookies in a box with at most one chocolate chip. [3]

(iv) The baker needs to add n more chocolate chips per batch of cookie dough in order to be at least 95% certain that a randomly chosen cookie has at least two chocolate chips. Express this information as an inequality in n, and hence find the smallest possible integer value of n. [3]

[Turn Over

11 **(a)** In MCJC, 75% of the students are right-handed. Given that a student is left-handed, the probability that he takes mathematics as a subject is $\frac{4}{7}$. Find the probability that a student is left-handed and does not take mathematics. [2]

If half of the right-handed students take mathematics, determine if being right-handed and taking mathematics are independent events. [2]

(b) In a game, a player chooses a card from a complete deck of poker cards. If the card chosen is a picture card, the player wins the game. If the card chosen is any number card other than the Ace, the player loses the game. If an Ace card is chosen, the player puts the card aside and takes a second card. He wins if the second card is a picture card, loses if it is a number card, and picks a third card if it is an Ace. He continues to do this until he either gets a picture card, in which case he wins, or a number card and he loses.

Find the probability that

(i) a player wins the game on picking the second card, [2]

(ii) a player wins the game, [2]

(iii) only one card is chosen, given that the player wins the game. [2]

[A deck of poker cards is made up of 4 Aces, 12 picture cards (Jacks, Queens and Kings) and 36 number cards.]

12 The sales manager of a retailer of KSI sunblock is monitoring the effects of its television advertising campaign. Over the last 7 weeks, different durations of television air time, x minutes, were used and the corresponding number of sales, y, in hundreds of bottles of KSI sunblock, were recorded as follows:

x	8	11	15	18	20	22	25
y	11.1	15.2	19.9	22.8	24.1	25.0	26.1

(i) Draw a scatter diagram for the data, labelling the axes clearly. [1]

(ii) Using the scatter diagram, explain if each of the following can model the given data set.

(A): $y = a + bx^2$, where a and b are positive constants

(B): $y = a + b\ln x$, where a is negative constant and b is positive constant

(C): $y = a + \frac{b}{x}$, where a and b are positive constants. [3]

Calculate the least squares estimates of a and b, and the product moment correlation coefficient for the model that best fits the data set. [2]

(iii) Give an interpretation of b in the context of this question. [1]

(iv) The advertising costs \$2000/min and each bottle of KSI sunblock yields a profit of \$20. Estimate the weekly profit when 10 mins of TV time is purchased. Comment on the reliability of the estimate. [3]

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