ANGLO-CHINESE JUNIOR COLLEGE
MATHEMATICS DEPARTMENT

MATHEMATICS
Higher 2

9740 / 01

Paper 1

18 August 2015

JC 2 PRELIMINARY EXAMINATION

Time allowed**: 3 hours**

Additional Materials: List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Index number, Form Class, graphic and/or scientific calculator model/s on the cover page.
Write your Index number and full name on all the work you hand in.
Write in dark blue or black pen on your answer scripts.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in the question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
At the end of the examination, fasten all your work securely together.

This document consists of **6** printed pages.

Anglo-Chinese Junior College

ANGLO-CHINESE JUNIOR COLLEGE
MATHEMATICS DEPARTMENT
JC2 Preliminary Examination 2015

MATHEMATICS 9740
Higher 2
Paper 1

/ 100

Index No: | | | | |

Form Class: ____________

Name: ____________________________

Calculator model: ____________________

Arrange your answers in the same numerical order.

Place this cover sheet on top of them and tie them together with the string provided.

Question No.	Marks
1	/3
2	/5
3	/6
4	/6
5	/8
6	/6
7	/9
8	/8
9	/9
10	/9
11	/10
12	/12
13	/9

Summary of Areas for Improvement				
Knowledge (K)	Careless Mistakes (C)	Read/Interpret Qn wrongly (R)	Formula (F)	Presentation (P)

1 Use the substitution $u = 3 - x^2$ to find $\int x^3 \sqrt{\left(3 - x^2\right)}\,\mathrm{d}x$. [3]

2 Using an algebraic method, solve the inequality $\dfrac{x+3}{1-x} \geq \dfrac{x}{2x+1}$. [3]

Hence solve the inequality $\dfrac{3-|x|}{1+|x|} \geq \dfrac{|x|}{2|x|-1}$ [2]

3 *ABCD* is a rectangular field whose sides, *AB* and *BC*, measure 2*a* m and *a* m respectively. A road runs along the side *AB*. A man, starting from *A*, wishes to reach the opposite corner *C* in the shortest possible time. He can walk along the road at 100 m per minute and across the field at 60 m per minute. Find an expression for the time, in minutes, he will take if he walks along the road to *P*, a point *x* m from *B*, and then across the field from *P* to *C*. [2]
Use differentiation to find, in terms of *a*, the value of *x* for the time taken to be the shortest possible. Find, also, the shortest possible time taken, and prove that it is the minimum. [4]

4 In triangle *ABC*, *AC* = 1, *BC* = 3 and angle *CAB* = θ radians.

(i) Show that $AB = \cos\theta + \sqrt{\left(9 - \sin^2\theta\right)}$. [3]

(ii) Given that θ is a sufficiently small angle, show that $AB \approx a + b\theta + c\theta^2$, for constants *a*, *b* and *c* to be determined. [3]

5 Given that $\left(1+x^2\right)\dfrac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{\tan^{-1}x}$, where $\tan^{-1}x$ denotes the principal value, and that *y* = 1 when *x* = 0, show that $\left(1+x^2\right)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = \left(1-2x\right)\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [2]

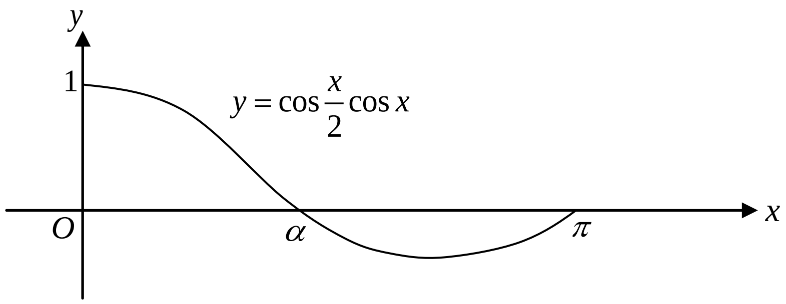
By repeated differentiation of this result, find the Maclaurin series for *y*, in ascending powers of *x*, up to and including the term in x^3. [3]

Verify that $y = \mathrm{e}^{\tan^{-1}x}$ is a solution of the differential equation $\left(1+x^2\right)\dfrac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{\tan^{-1}x}$. [1]

Show that the series expansion for $\mathrm{e}^{\tan^{-1}x}$, up to and including the term in x^3, can be expressed as $\mathrm{e}^{\tan^{-1}x} = \mathrm{e}^x + kx^3$, where the numerical value of *k* is to be determined. [2]

[Turn Over

6



The diagram shows the curve with equation $y=\cos\frac{x}{2}\cos x$ for $0\le x\le\pi$. The curve crosses the y-axis at $y=1$ and the x-axis at $x=\alpha$ and $x=\pi$.

(i) Find

$$\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}}\left|\cos\frac{x}{2}\cos x\right|\,\mathrm{d}x,$$

leaving your answer in the form $a+b\sqrt{2}+c\sqrt{3}$, where a, b and c are rational numbers to be determined. [5]

(ii) Explain why $\left|\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}}\cos\frac{x}{2}\cos x\,\mathrm{d}x\right|$ is smaller than your answer in **(i)**. You may make reference to the graph. [1]

7 A sequence u_1, u_2, u_3, … is given by

$$u_1=-2 \quad \text{and} \quad u_n=u_{n-1}+\ln\left(1+\frac{2n-1}{(n-1)^2}\right)-2 \qquad \text{for } n\ge 2.$$

(i) Use the method of mathematical induction to prove that for all positive integers n,

$$u_n=2(\ln n-n).$$ [4]

(ii) Hence find $\sum_{r=15}^{n}\frac{\mathrm{e}^{u_r}}{r^2}$ in terms of n. [3]

(iii) Give a reason why the series $\sum_{r=1}^{\infty}\frac{\mathrm{e}^{u_r}}{r^2}$ converges, and write down its exact value. [2]

8 In order to model a particular predator-prey relationship, a biology student came up with the following differential equations:

$$\frac{\mathrm{d}x}{\mathrm{d}t}=1-\frac{x}{100} \qquad \text{(A)}$$

$$\frac{\mathrm{d}y}{\mathrm{d}t}=x-100 \qquad \text{(B),}$$

where the variables x and y denote the number (in thousands) of predator and prey respectively, t days after the start of the observation. There were 50 000 predators at the start of the observation.

(i) By solving equation (A), show that $x=100-k\mathrm{e}^{-0.01t}$, where k is a constant to be determined. [4]

(ii) What can you say about the population of the predator after several years? [1]

(iii) In the long run, the model shows that number of prey approaches 5 million. Using your answer in **(i)**, find y in terms of t. [3]

9 The curve C is given by the equation $y = \frac{1}{x} + \frac{2}{x^2}, x \neq 0$.

(i) Without using a calculator, find the set of values that y can take. [2]

(ii) Sketch the curve C, stating the equations of any asymptotes and the coordinates of any turning points and points of intersection with the axes. [3]

Given that the solution of the inequality $ax^2 + bx + c > \frac{1}{x} + \frac{2}{x^2}$ is the set

$$\{x \in \mathbb{R} : -1.5 < x < -1 \text{ or } 1 < x < k\},$$

find the values of a, b and c. [3]

Hence find the value of k. [1]

10 The points A and B have coordinates $(-1,\ 3,\ 3)$ and $(1,\ k,\ 5)$ respectively, where $k \in \mathbb{R}$.

(a) **(i)** Find the length of projection of $\overrightarrow{OB}$ on $\overrightarrow{OA}$ in terms of k. [2]

(ii) State the value of k that gives the shortest length of projection. State also the relationship between $\overrightarrow{OA}$ and $\overrightarrow{OB}$ at this value of k. [2]

(b) Let $k = -2$. The point C is the reflection of the origin O in the line AB.

(i) Find the position vector of C. [3]

(ii) Find the exact area of the quadrilateral $OACB$. [2]

11 The complex numbers p and q are given by $k + 2\text{i}$ and $3 - 3\text{i}$ respectively, where $k \in \mathbb{R}$, $k > 0$.

(a) $\text{P}(x)$ is a polynomial of degree n with real coefficients where the coefficient of x^n is 1. Given that p and q are roots of $\text{P}(x) = 0$, state the least possible value of n. For this value of n, express $\text{P}(x)$ as a product of quadratic factors with real coefficients. [3]

(b) **(i)** The complex number $\frac{\text{i}q^2}{2p}$ has modulus $\frac{9}{4}$ and argument α, where $-\pi < \alpha \leq \pi$. Without using a calculator, find the exact values of k and α. [4]

(ii) Solve the equation $\text{i}z^4 = -\frac{q^2}{2p}$, expressing your answers in the form $r\text{e}^{\text{i}\theta}$ where $r > 0$ and $-\pi < \theta \leq \pi$. [3]

12 The curve C has parametric equations $x = \frac{u}{1+u}$, $y = \frac{u^2}{1+u}$, where $u \neq -1$.

(i) Express $\frac{\text{d}y}{\text{d}x}$ in terms of u. [2]

(ii) Given that u is increasing at a rate of 2 units per second, find the rate at which $\frac{\text{d}y}{\text{d}x}$ is increasing when $u = 1$. [2]

(iii) Find the acute angle between the tangent at $x = 0.5$ and the normal at $u = -0.5$. [4]

(iv) The distinct points P and Q on the curve have parameters p and q respectively. If the tangents at P and Q intersect the y-axis at the same point, show that $p + q = 0$. [4]

[Turn Over

13 The owner of a newly opened café decided to rent a painting from an artist as part of the decoration of the café. They set about drafting up a contract for the terms of the rental.
The artist proposed a rental contract (Version 1) stating that the owner will pay the artist \$15 for the 1st day of rental and for each subsequent day, the daily rental cost will increase by \$0.50.

(i) On which day of the rental will the owner first have to pay the artist more than \$39 as the daily rental rate? [2]

The owner proposed an alternative contract (Version 2), where the daily rental rate is such that on the *n*th day of the rental, the amount of money, in dollars, the owner has to pay to the artist is given by the function

$$\mathrm{f}(n) = \frac{12000}{4n^2 + 4n - 3}.$$

(ii) Express $\mathrm{f}(n)$ in the form $\dfrac{A}{2n-1} + \dfrac{B}{2n+3}$, where A and B are constants to be determined. [1]

(iii) Hence show that with Version 2 of the contract, the total amount of money the artist will receive at the end of m days of rental is

$$4000 - \frac{12000(m+1)}{(2m+1)(2m+3)}.$$ [3]

(iv) The artist accepted Version 2 of the contract, and terminated the contract at the end of k days. Given that the artist received more money in total from Version 2 than if he had chosen Version 1, find the largest possible value of k. [3]

- End of Paper -